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SOME ASPECTS OF VARIANCE SPECTRA OF SYNOPTIC SCALE TROPOSPHERIC WIND COMPONENTS IN MIDLATITUDES AND IN THE TROPICS

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ABSTRACT

The primary concern of this paper is an examination of the utility of spectral techniques in detecting wave modes of the large-scale tropospheric circulation. Sample time- and space-spectra of meridional winds in middle latitudes are presented. A maximum in the spatial spectra of the transient waves at hemispheric wave numbers 5–7 is shown, in agreement with similar, previously published spectra. The temporal spectra, unlike the spatial spectra, fail to show any significant features that can be attributed to the passage of baroclinic disturbances. (In this paper, the frequency range considered is limited to 0.03–0.50 per day.) The continuum of the coherence statistic between lower and upper tropospheric wind components is highly significant, but no significant "peaks" or maxima exist.

Because of recent interest in the dynamics of tropical regions, spectral techniques are employed on meridional wind data for five tropical stations with record lengths of from 3 to 7 yr. At 850 mb, the two stations in the west-central Pacific possess spectra with a peak in the 4–5-day period range. Similar spectra for two stations in the Atlantic Ocean and one in Indonesia do not exhibit this feature, and at 200 mb none of the five spectra has any indication of a significant peak at any frequency. The coherence between the two levels is not significant at any frequency, with the possible exception of the 0.20–0.25 per day band at the Pacific stations. As in the case of the midlatitude spectra, the peak in the spatial spectra of the transient 200-mb meridional winds does not possess a counterpart in the temporal spectra.

Finally, complex demodulation is used to investigate possible nonstationary aspects of the 4–5-day period wave mode for the Pacific stations. The results suggest nonstationary behavior with an annual modulation but of a sporadic nature. In addition, the demodulation procedure provides information on the questionable significance of the coherence between the lower and upper tropospheric tropical meridional winds.

1. INTRODUCTION

Variance spectrum analysis has been used by many researchers as a tool in investigating large-scale, synoptic wave motion in middle latitudes. Recently, the use of such analyses has been extended to upper wind time-series in the Tropics. The work of Yanai et al. (1968), Wallace and Chang (1969), and Chang et al. (1970) has provided evidence of dominant wave frequencies in the temporal spectra of winds in the troposphere and in the lower stratosphere.

The purposes of my contribution are to examine critically the use of spectral techniques for detecting wave modes in the large-scale circulation, to add to the inventory of temporal and spatial spectra of tropical winds and compare these with spectra from extratropical regions, and to present some evidence for a type of nonstationary behavior in the temporal spectra of wave motion in the Tropics.

For purposes of discussion, I will restrict myself to the view that the purposes of performing spectral analysis are to verify the existence of (1) dominant wave modes or scales of motion and/or (2) spectral structure obeying power laws or similar predicted behavior. Almost all existing studies can be placed in either of these categories.

Two methodologically differing approaches can be and have been employed in accomplishing these purposes. The first approach is to utilize sample spectra from a set or sets of observations to verify conclusions based upon theory. Statistical techniques are used to either accept or reject a hypothesis made on the basis of a priori knowledge. An elegant example of this type of application of spectrum analysis is that of Platzman and Rao (1964). The second approach is to obtain a set or sets of observations and to perform spectral analyses for heuristic or diagnostic purposes. In this instance, statistical hypotheses are seldom if ever formulated and tested, usually because a priori knowledge of the physics or dynamics producing the series is not sufficient. In the event that statistical tests are employed, special ones must be used, and the a posteriori interpretation of statistical results is fraught with pitfalls. The use of statistical tools for essentially nonstatistical purposes seems to me to represent a somewhat less than satisfying methodology.

Specific examples of this latter use of spectrum analysis will not be given, but one general area of endeavor can be mentioned. The best known example is "cycle hunting." The literature contains many efforts at finding peaks in spectra when no a priori reason to expect a peak at a particular frequency or at any frequency exists. A revealing summary of the results of such efforts has been given by Berlage (1957). Statistical theory stipulates that peaks may indeed be expected in sample spectra and that conventional tests may not be used in a posteriori type of analysis.

The problem of evaluating the statistical significance of a peak at an unpredicted frequency is fundamentally different than that of an a priori expected peak. This

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problem has been discussed, for example, by Mitchell et al. (1966). Briefly, if there are q independent spectral estimates obtained over the resolvable frequency range and the 95-percent prior confidence level is chosen, approximately $0.05q$ estimates can be expected to exceed the confidence level specified by the distribution of χ^2/ν (ν=number of degrees of freedom). Prior sampling theory does not specify at what frequencies these estimates will occur and posteriori acceptance of them as significant peaks clearly violates statistical reasoning. The problem of proper posteriori sampling limits is not easily solved; two approaches are used in this study. The first, an excellent statistical rule in general, is to confirm unexpected spectral features with independent data samples. The second, as described in the aforementioned reference by Mitchell, is to use prior significance levels that are higher than conventionally employed. For example, if, instead of the 95-percent level, the 99.9-percent level is used, then $0.001q$ estimates are expected to exceed that level. If $q=100$, then in 100 such sample spectra, 10 spectral estimates can be expected to exceed the chi-square over degrees of freedom ratio; thus we can designate the 99.9-percent prior limits equivalent to 90-percent posteriori limits. The calculation of the 95-percent posteriori limit will, in general, require interpolation in the tables of chi-square.

A complicating feature of many, if not all, of meteorological time (or space) series is that they are not statistically stationary. Owing to diurnal, seasonal, regional, and, possibly, secular changes in the governing physics and dynamics of the atmosphere, nonstationary behavior may be expected. The type of nonstationarity, however, may not always be evident. Conventional tests of significance for sample spectra assume that the series is extracted from a stationary process and must therefore be used and interpreted with care—particularly in the event of posteriori "discovery" of spectral features. Some statistical techniques are available to investigate nonstationary behavior. An example of one such technique is given in section 5.

2. REVIEW OF SPATIAL AND TEMPORAL SPECTRA OF WIND COMPONENTS

The four-dimensional flow field of the atmosphere has been, almost without exception, examined with spectral techniques by calculating one-dimensional spectra. Either single-station time series are used to obtain spectra in terms of temporal frequency or spatial representations (synoptic maps) of the velocity fields are employed to give spectra with spatial frequency, or wave number, as the independent variable. A few examples exist in the literature of combined space-time spectra (Shapiro and Ward 1960, Kao 1970).

The surprisingly few published Eulerian-time [2] spectra of observed tropospheric wind components in the middle latitudes suggest that, aside from the diurnal and annual components, no feature consistently appears that can be interpreted as a peak representing a periodicity or quasi-periodicity (Chiu 1960). These spectra show a smooth continuum background decreasing from low to high frequencies with superimposed peaks and valleys representing sampling variations. This type of spectrum has been dubbed a red-noise spectrum, invoking an analogy with optical frequencies, because its shape is such that maximum values of the background spectrum are at the lowest frequencies (Gilman et al. 1963). It should be emphasized that the term spectral peak as it is used here is not a feature of the background or continuum spectrum, but denotes a distinguishable feature no matter how the spectrum is transformed for display. In addition, care must be taken in the interpretation of what is meant by a maximum in the background spectrum. For a discussion of these matters the reader is referred to correspondence by Julian (1966), Chiu and Crutcher (1966), and Chiu (1967).

A survey of Eulerian-space spectra of middle-latitude wind components given in the literature indicates that, in contrast to the time-spectra, dominant spatial scales do exist (Van Mieghem 1961). In particular, spectra of the meridional wind components show that a maximum exists at hemispheric zonal wave numbers 5 and 6. If the analysis is restricted to the transitory flow field, this maximum is quite pronounced (Saltzman and Fleisher 1962). The spectra of the zonal components do not, however, possess a comparable maximum; instead, the spectra decrease monotonically with increasing wave number.

Figures 1 and 2 show, respectively, the Eulerian-time spectra of the meridional wind at a middle-latitude station and a Eulerian-space spectra of the same component. The former was estimated from approximately 9 yr of rawinsonde data from Columbia, Mo. (39°N, 92°W), while the latter, using observed winds at 50°–52°N, was taken from data analyzed by Julian et al. (1970). The details of estimation of the time-spectra are given in the appendix and of the space-spectra in Julian et al. (1970).

The figures emphasize the points made in the paragraphs above. None of the features in the time-spectra is statistically significant using a posteriori test that uses the prior significance level of 99.9 percent, corresponding roughly to a 97-percent posteriori limit.

The maximum in the spatial spectra is highly significant either on a posteriori basis or on an a priori basis assuming that the maximum is a result of baroclinic instability and specifying that it should appear in the wave number 6–8 region.

In principle, the combination of Eulerian-time and -space spectra of wind components can be used to investigate properties of wave motion in the atmosphere. Wave theory usually produces a dispersion equation of the form

$$f=\lambda^{-1}U\pm f_t(\lambda, \ldots) \quad (1)$$

[2] This term is used to distinguish this form of time-series spectra from that which employs a Lagrangian coordinate system

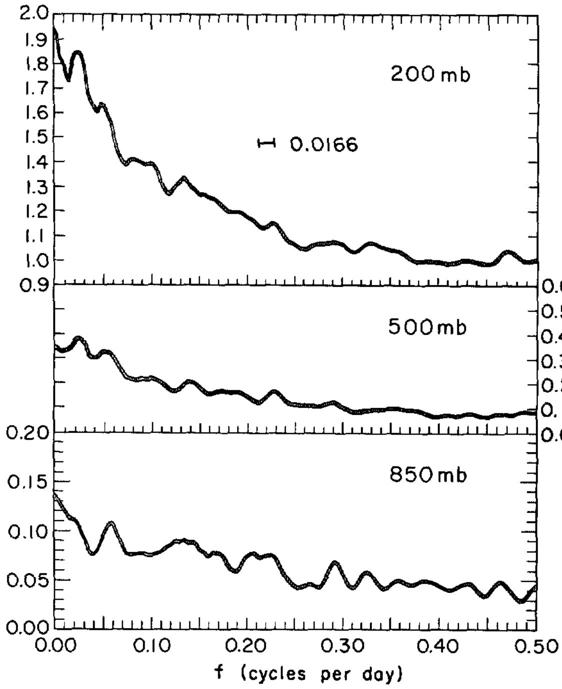

FIGURE 1.—Variance spectra of the meridional component of the wind at Columbia, Mo. (39°N, 92°W), for three pressure levels. The abscissa is linear frequency with the Nyquist frequency equal to 0.5 day^{-1} and the ordinate is m^2 · s^{-2} · bandwidth^{-1}; the bandwidth of the spectra is $\Delta f=0.0166$ day $^{-1}$. The ordinate scales are not identical for the three spectra. The sampling limits (chi-square divided by the degrees of freedom) are: 5 and 95 percent prior, 0.77 and 1.25, and 99.9 percent prior ($\approx 97\%$ posteriori), 1.51.

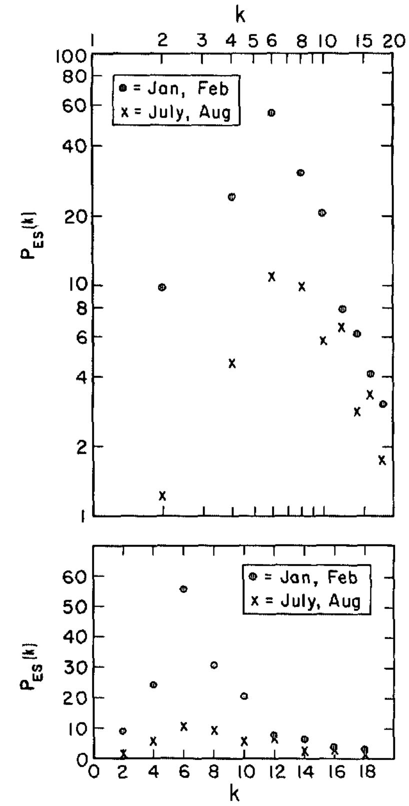

FIGURE 2.—Spatial (zonal wave number, k) spectra of the 200-mb meridional wind component along 50°–52°N estimated from observed rawin data. The top diagram presents the spectra in logarithm components, the bottom in linear coordinates. Both winter and summer data are included.

which describes the observed (Doppler shifted) wave frequency f as a function of the intrinsic frequency, f_i (a function of the wavelength λ), and mean zonal velocity, U. A simple transformation or rescaling of the spectra in terms of the variables λ (or k) and f is unlikely for a number of reasons. Atmospheric waves of synoptic scale are dispersive waves and cannot be expected to propagate without changing their apparent frequency and shape. Vinnichenko (1970) has suggested using the group velocity, C_g, instead of the phase velocity, C, in effecting a transformation of time- and space-spectra. It seems to me that although a temporal frequency derived from C is clearly inappropriate for such a transformation, one derived from C_g is likewise inappropriate. Examination of Hovmöller charts (Hovmöller 1949), for example, indicates that, as Rossby-type waves pass over a station, the meridional component of the motion is influenced both by a wave velocity on the order of C and by a group velocity.

There are, however, a number of additional reasons why a simple transformation between time- and space-spectra cannot be expected. The mean zonal velocity U in eq (1) cannot in fact be expected to be constant in space and time and, therefore, the intrinsic wave frequency will be Doppler-shifted over some range of observed frequency. Also, if the propagation of the Rossby-type waves is not in a purely zonal direction, the group velocity vector will not be exactly opposite to the direction of the wave velocity (Platzman 1968). And, of course, real Rossby-type waves in the atmosphere are of a nonlinear nature and the use of dispersion equations such as eq (1) involves linear theory. Without doubt, at the time scales of weeks to months, interaction between the wave and the zonal flow must be expected.

The main point of this discussion is to emphasize that in middle latitudes we have a body of linear theory which allows a prediction of, first, approximately where the maxima in the spatial spectra of the synoptic scale motion fields should occur and, second, how the spatial and temporal spectra should be related. However, in this case it is not a simple matter to account for the shape of the *observed* spectra. The maximum so evident in the spectra of the meridional wind component of scales of about 4000–5000 km has no detectable counterpart in the time-spectra. Qualitatively, this spatial maximum must be said to be "smeared out" over a very broad range of temporal frequency by virtue of the fact that the gravity-inertial waves of middle latitudes are nonlinear and dispersive, and that the current on which they are superimposed also fluctuates.

It is of major interest to ascertain the characteristics of the velocity-spectra in tropical regions because a modest body of theory comparable to that for midlatitude syn-

TABLE 1.—*Summary of data analyzed and characteristics of spectral analysis*

Station	Lat., long.	Dates used	Length (days)	Percent Interpolated 850 mb	Percent Interpolated 200 mb	Band-width	Dof [a]
Singapore	(1°S, 104°E)	Jan. 5, 1961–June 24, 1964	1,264	3.0	7.9	0.0166	39
		Feb. 12, 1965–Dec. 31, 1968	1,408	0.8	8.2	.0163	40
Kwajalein	(9°N, 168°E)	Mar. 8, 1954–Sept. 7, 1957	1,268	5.5	13.4	.0164	39
		Nov. 21, 1958–May 23, 1962	1,280	6.0	6.7	.0164	39
Canton Island	(3°S, 172°W)	Do.	1,280	1.7	2.4	.0164	39
		Apr. 20, 1962–Oct. 20, 1965	1,280	2.6	5.5	.0164	39
Balboa	(9°N, 80°W)	June 1, 1957–Oct. 25, 1958	512	6.8	4.3	.0215	19
		Apr. 1, 1966–Aug. 25, 1967	512	1.4	1.6	.0215	19
Ascension Island	(8°S, 15°W)	Nov. 21, 1958–May 23, 1962	1,280	1.8	1.0	.0164	39

[a] Degrees of freedom

optic systems is beginning to appear for tropical motions (Matsuno 1966, Lindzen 1967). Observational evidence seems to indicate that the velocity spectra possess features that midlatitude velocity spectra do not.

3. TIME-SPECTRA IN THE TROPICS

Recent analyses of rawinsonde observations made in tropical latitudes by various investigators (summarized by Wallace 1969) have provided evidence of a mode of transitory wave motion with an apparently narrow-band maximum in the spectra of meridional wind. The range of period of this maximum is about 4–5 days. The appearance of such a maximum is quite interesting because no comparably "tuned" feature appears in middle latitude spectra and, from what I can determine from the literature, its existence at those particular frequencies was not anticipated.

To investigate more of the statistical behavior of this spectral feature, I have computed spectra of the zonal and meridional component of rawinsonde-observed winds and cospectra of these components at the same and different levels. To keep the computational task within manageable limits, one level representative of the lower troposphere (850 mb) and one representative of the upper troposphere (200 mb) were chosen.

An effort was made to acquire wind observations from as many stations in the Tropics as was possible providing that the length of record was on the order of years. Table 1 summarizes the data sets used: the stations selected were Canton; Kwajalein; Singapore; Balboa, Canal Zone; and Ascension Island. The objective was to allow two independent data sets per station, each of which extended over at least 2 yr. This objective was achieved for Singapore, Kwajalein, and Canton but because of gaps in the records available it was not achieved for Ascension and Balboa, Canal Zone. It should be emphasized that the intervals covered by the data were governed solely by what data were available at the time the study commenced. The exact record length in each case was varied slightly to achieve a number easily factorable for efficiency in running the fast Fourier transform. Data were interpolated when missing; generally, this was done by referring to observations taken within 12 hr. In a few cases (Singapore and Kwajalein at 200 mb), the percentage of interpolated data is relatively high (6th column of table 1). For the latter station, spatial interpolation using nearby stations was performed whenever possible. Unless the missing observations bear some relation in time to the phase of waves with a certain frequency, the spectra should not be affected adversely by the interpolated data. I do not believe such a relation is likely.

Figures 3–4 and 5–6 present the 850- and 200-mb meridional wind spectra, respectively, for the five tropical stations. In all these spectra, the estimates are plotted from a low frequency of 0.03 per day to the Nyquist frequency of 1/2 per day. This omission of the very lowest frequencies was intended to avoid the contamination of the spectral estimates by the annual and semiannual variations in the wind field. Also, some aspects of the spectra in this low frequency range will be covered in another contribution (Madden and Julian 1971). The use of the fast Fourier transform and the estimation scheme outlined in the appendix provides a convenient mechanism for omitting portions of the spectrum where periodic components occur, or, if desired, analyzing these portions with a variable spectral window. The ordinate scale in each case is in units of $m^2 \cdot s^{-2} \cdot$ bandwidth^{-1}, with the bandwidth in each case marked.

For the 850-mb level, only those spectra for Canton and Kwajalein provide evidence for a peak in the 1/4–1/5 per day region. All of these features are significant on an a priori basis at the 5-percent level. Taken together with the work previously cited, these spectra provide overwhelming support for a wave mode with that temporal frequency in the tropical Pacific. However, except for a feature at the proper frequency on only one of the two spectra for Singapore (which is significantly above the background spectrum at the prior 5-percent level), there is no evidence for the existence of such a mode in the tropical Atlantic or in the Indonesian region.

At 200 mb, none of the spectra has a local maxima in the 1/4–1/5 per day frequency band that is significant in any sense. Because the amount of data shown here is orders of magnitude larger than that used by Yanai et al. (1968) and allows a much smaller signal-to-noise ratio to be detected if, in fact, any such feature were in the data, I conclude that the peak at the upper tropospheric levels obtained by those workers must have been a sampling variation and not a feature of the population spectrum. There is a possibility that a wave mode with this frequency is a nonstationary or sporadic feature of the upper troposphere in the central tropical Pacific. However, if so, it should have survived in some fashion the averaging process inherent in using records on the order of 3–7 yr in

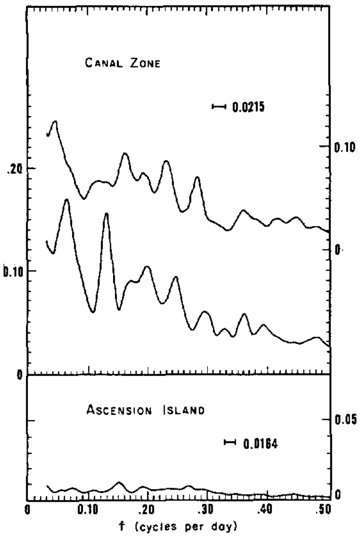

FIGURE 3.—Meridional wind spectra (850 mb) for the stations indicated. The curves are for the intervals indicated in table 1 with the earlier interval the top curve and the later the bottom curve. The ordinate is in units of $m^2 \cdot s^{-2} \cdot$ bandwith^{-1}.

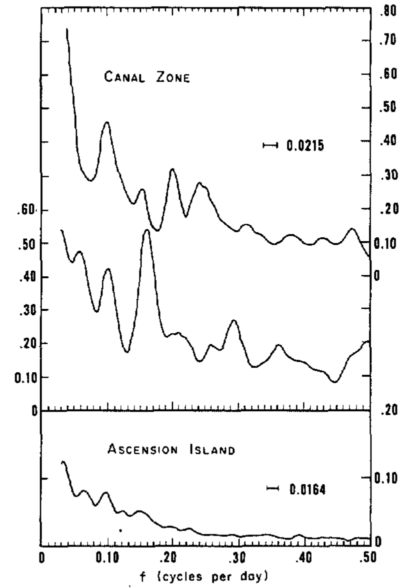

FIGURE 5.—Same as figure 3 except for 200-mb meridional wind.

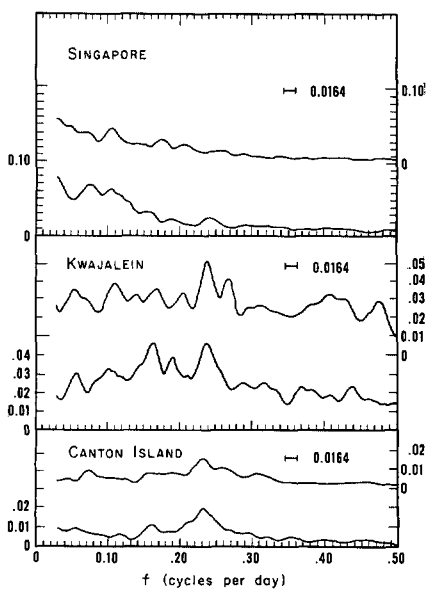

FIGURE 4.—Same as figure 3 for the stations indicated, except that for Canton Island the two spectra shown are for the intervals Apr. 1962–Oct. 1965 (top) and Nov. 1958–May 1962 (bottom).

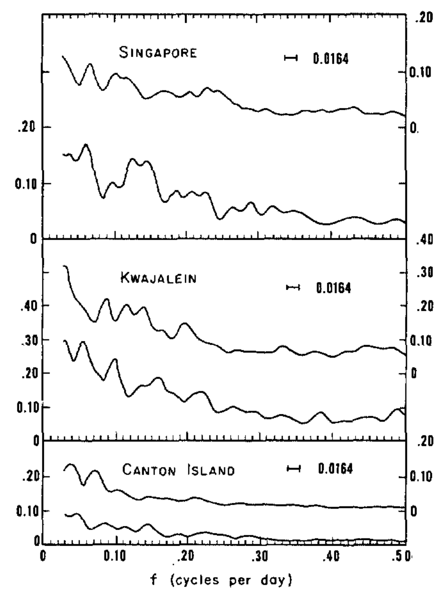

FIGURE 6.—Same as figure 4 except for 200-mb meridional wind.

length. Apparently, frequencies on either side of the 1/4–1/5 per day band also sporadically possess maxima, and we may legitimately ask whether these are also characteristic of inertial-gravitational wave modes or whether the entire time-spectrum of the meridional wind in the upper troposphere of the Tropics is not similar to that in middle latitudes, namely, simply a red-noise spectrum.

Some evidence has appeared that suggests that the meridional wind components in the upper and lower troposphere are coherent for the 1/4–1/5 per day frequency band (Yanai et al. 1968, Chang et al. 1970). The coherence-squared statistics were computed for the 850- and 200-mb v-series for all stations. Figures 7 and 8 present the results. A similar computation for Columbia, Mo., station is in figure 9—here 300 mb was selected as a representative upper tropospheric level.

In assessing the significance of the coherence-squares, I will accept the earlier finding of maximum coherence in the 1/4–1/5 per day interval and use prior significance levels for this band. For the rest of the frequency range however, posteriori limits will be used. Using the material on posteriori limits outlined in section 1, the approximate 5-percent posteriori limit is calculated to be equal to the 0.2-percent prior limit for all stations. If one employs the 5-percent prior and the 0.2-percent prior limits to the distribution of the coherence-squared statistic for the

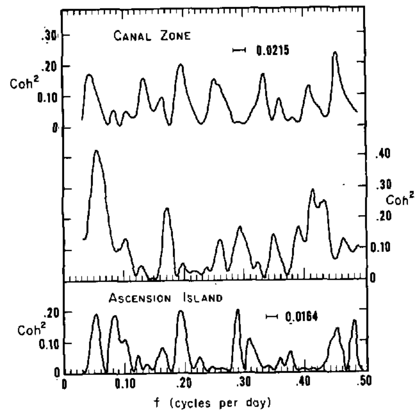

FIGURE 7.—The coherence-squared statistic for the 850- and 200-mb meridional winds for the stations indicated. For Balboa, Canal Zone, the 95-percent prior sampling limit (on the null hypothesis of no relationship) is 0.28, and the 99.9-percent prior limit (roughly equivalent to the 98% posteriori limit) is 0.54. For Ascension Island, the 95-percent prior sampling limit is 0.15 and the 99.9-percent prior limit (roughly the 97% posteriori limit) is 0.31.

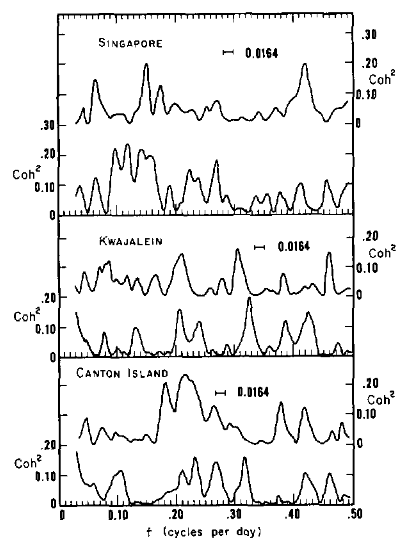

FIGURE 8.—Same as figure 7 for the indicated stations. The limits are the same as for Ascension Island in figure 7.

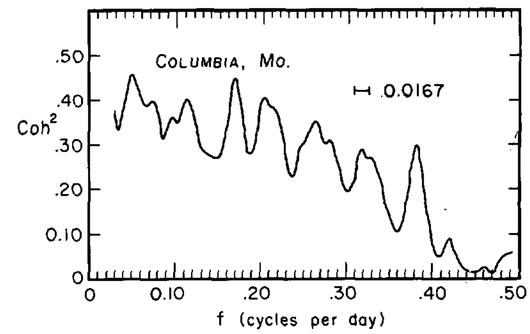

FIGURE 9.—The coherence-squared statistic for the 850- and 300-mb meridional wind components, Columbia, Mo. The 99.9-percent prior (97% posteriori) sampling limit on the null hypothesis of no relationship is 0.13.

tropical stations (figs. 7 and 8), it is clear that the only peaks exceeding the appropriate limits are those in the 1/4–1/5 per day band at Kwajalein and Canton. An important consideration here, however, is that the acceptance of these peaks as significant depends entirely upon the use of the prior sampling limit: in three of the four plots for these two stations, there are peak values of the coherence-squared which exceed those in the 4–5-day period range. This fact places considerable dependence upon the previous analyses and produces the following comments. If, indeed, the upper and lower troposphere in the tropical Pacific are coherent in the 1/4–1/5 per day interval, the relationship must be either (1) extremely weak and near the noise level or (2) nonstationary in the sense that the coherence is only sporadically strong and nonexistent the remainder of the time. Material presented in section 5 will bear on this latter possibility.

The Columbia, Mo., record was included to illustrate the coherence in the vertical of the meridional wind component for middle latitudes. Figure 9 portrays an obvious, highly significant, continuum in coherence for all frequencies except those close to the Nyquist frequency of 1/2 per day. The fluctuations are sampling variations superimposed upon this continuum. It should be noted that, in this case, the use of a sampling limit of coherence calculated on the null hypothesis of zero population coherence is valid only for assessing the significance of the continuum. Testing of the significance of individual peaks and troughs in restricted frequency intervals would require the establishment of the population continuum about which the sampling limits apply. In the case of the work of Yanai et al. (1968) and Chang et al. (1970), it should be pointed out that only the significance of the coherence statistic on the null hypothesis of zero population coherence was established—no tests were carried out on the significance of the rise in coherence in the 1/4–1/5 per day band above the background level of coherence. Figures 7 and 8 suggest that the background continuum coherence might well be nearly zero making the above point moot, but I make it nevertheless.

Figures 10–11 and 12–13 give the variance spectra of the zonal component of the wind at 850 and 200 mb, respectively. At 850 mb, there is no convincing evidence for peaks at any of the five stations with the possible exception of the features at a frequency of 0.12–0.13 per day at Kwajalein. Neither of the peaks is significant at a posteriori 5-percent level. However, because they do occur on both spectra at approximately the same frequency, the degree of confidence assigned these features must be increased. On the other hand, Canton's spectra show no evidence of peaks at this frequency. Thus, in the absence of any rationale for expecting a peak at this frequency and of any evidence that the zonal motion involved is anything but a local phenomenon at Kwajalein, judgement as to the significance of this spectral feature must be reserved.

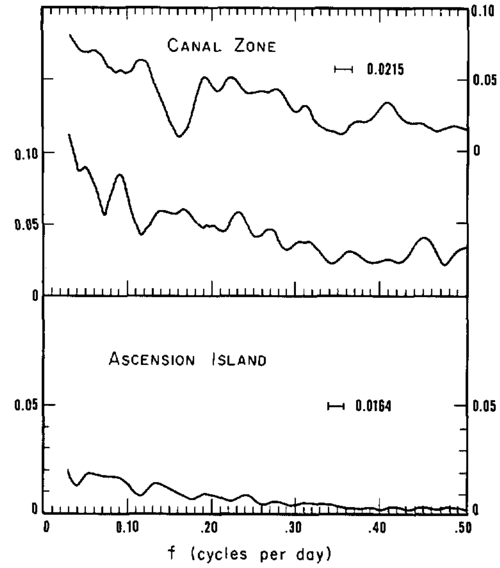

Figure 10.—Same as figure 3 except for 850-mb zonal wind.

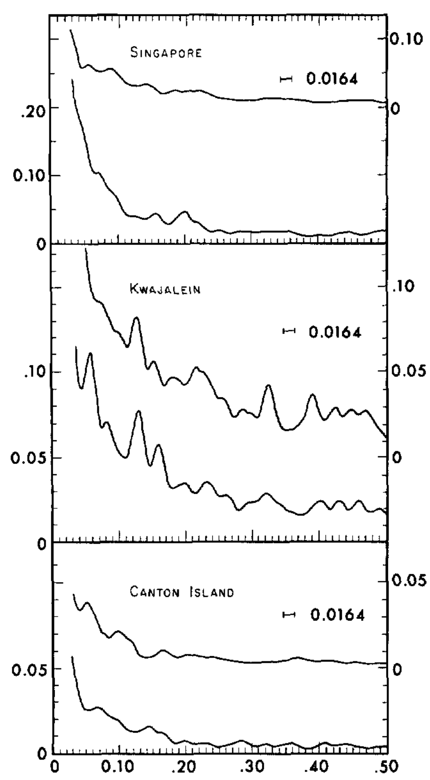

Figure 11.—Same as figure 4 except for 850-mb zonal wind.

At the 200-mb level, there is no evidence at all of any feature that can be considered significant. The apparently large peak in the topmost spectrum for Balboa, Canal Zone, does not meet a 5-percent posteriori significance level. Furthermore, the other spectrum does not support this feature.

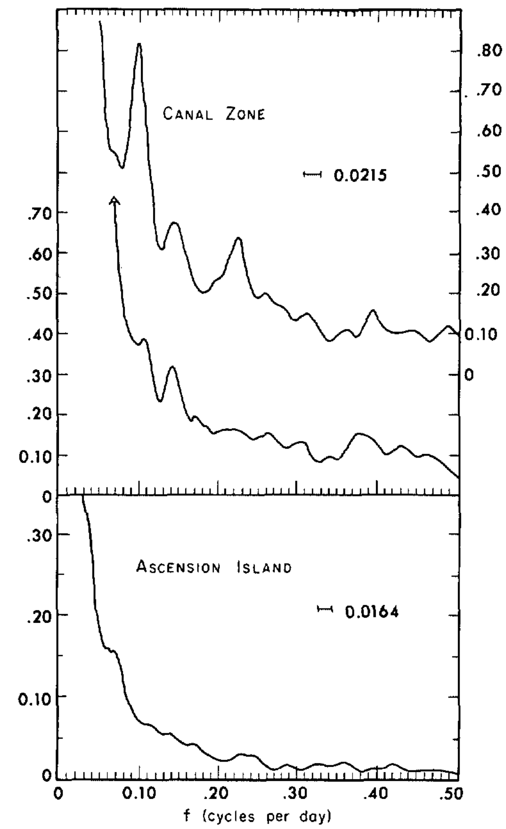

Figure 12.—Same as figure 3 except for 200-mb zonal wind.

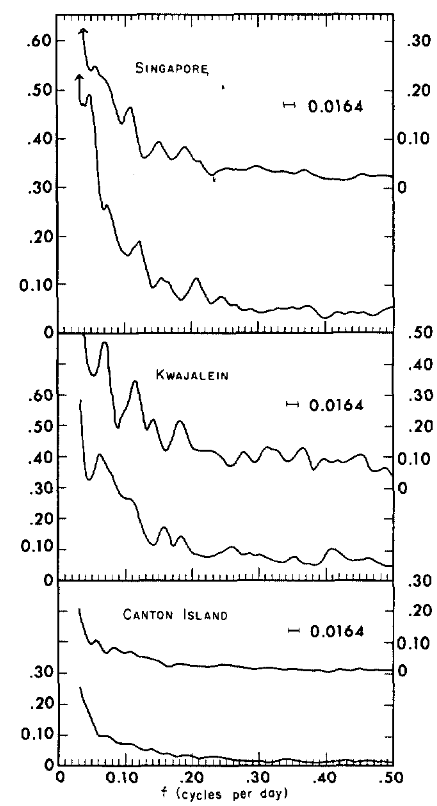

Figure 13.—Same as figure 4 except for 200-mb zonal wind.

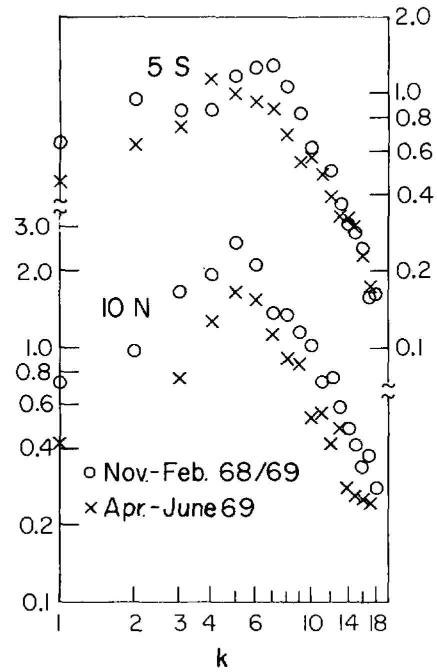

FIGURE 14.—Spatial spectra of the meridional wind component at 200 mb along 5°S and 10°N for the times indicated. The ordinate is amplitude squared ($m^2 \cdot s^{-2}$) and the abscissa is the zonal wave number. Data are from objectively analyzed wind component analyses for the Tropics (Bedient 1967).

In summary, examination of the zonal and meridional wind components at 850 and 200 mb at five tropical stations provides statistical support for only a single spectral feature—a 4–5-day period oscillation at 850 mb in the central equatorial Pacific. The upper tropical troposphere is apparently devoid of any dominant or distinct temporal scales.

4. SPATIAL SPECTRA IN THE TROPICS

The availability of objectively analyzed wind analyses for the Tropics enables component spectra in the domain of zonal wave number to be estimated. Figure 14 presents spatial spectra of the 200-mb meridional component of the wind along 5°S and 10°N. Two time intervals, November–February 1968–69 and April–June 1969, were used. The wind analyses were the twice-daily maps produced by the tropical analysis group at the National Meteorological Center. They include aircraft and satellite cloud-derived winds as well as conventional data—the analysis procedure has been described by Bedient (1967).

There is clearly evidence for a dominant spatial scale in these 200-mb v-spectra at zonal hemispheric wave numbers of $k=5$–9, or wavelengths of 8000 to 5000 km. Only the transient part of the meridional wind was utilized for these spectra; in each, the time-averaged or "standing" velocity component was removed. The dominant spatial scale indicated here is significantly larger than that for middle latitudes ($\approx$4000 km, fig. 2). However, as in middle latitudes, no evidence for a counterpart of the dominant spatial scale in the temporal spectra is apparent. The suggestion is that, in perhaps the same way that the spatial spectral peak in the extratropics is smeared out over a broad range of temporal frequency, the dominant spatial scale in the Tropics likewise is unresolvable on the temporal spectra. I suggest that the same meteorological factors are operating in both cases.

5. COMPLEX DEMODULATION OF MERIDIONAL WIND TIME SERIES

Complex demodulation is a form of time series analysis useful for studying the nonstationary behavior or nonlinear aspects of sample series. Discussion and some applications of the technique have been given by Granger and Hatanaka (1964) and Godfrey (1965). The object of complex demodulation is to estimate the time variation of the amount of variance (or power) in a particular frequency band of the spectrum.

Let $v(t)$ be the real-valued time series under investigation, and $S_v(f)$ its spectrum. Multiplication of the series by the complex exponential with a kernel containing the center frequency to be demodulated, f_c, gives

$$\tilde{v}(t, f_c)=v(t) \exp [-(i2\pi f_c t)]. \tag{2}$$

The complex-valued series $\tilde{v}(t, f_c)$ has thus been "frequency-shifted" in that the general frequency, f, of the original series is now frequency $(f-f_c)$, and the frequency demodulated now has a frequency of zero. This frequency-shifted series is now subjected to a low-pass filter operation:

$$v_*(t, f_c)=\sum_{l=-m}^{m} w(l)\tilde{v}(t+l, f_c) \tag{3}$$

where the $w(l)$ are a series of weights. The resulting time-series elements, called demodulates, are used to compute

$$S_*(t, f_c)=|v_*(t, f_c)|^2$$

and

$$\Phi_*(t, f_c)=\tan^{-1}\left(\frac{I_m v_*}{R_e v_*}\right). \tag{4}$$

The spectral demodulates, $S_*(t, f_c)$, give the time variation of the variance in a bandwidth centered on frequency f_c, and of width Δf determined by the low-pass filter weights. The phase function $\Phi_*(t, f_c)$ portrays the time variation in the phase of the oscillations of frequencies $f_c \pm \Delta f$ relative to some fiducial point. Whereas the spectrum $S_v(f)$ discards all phase information, $\Phi_*(t, f_c)$ allows an examination of the phase behavior of a portion of the spectrum.

The use of the fast Fourier transform (fFt) provides a computationally convenient and efficient algorithm for performing complex demodulation. Briefly, all Fourier amplitude coefficients are obtained for the entire series by the fFt. The coefficients are then shifted, or relabled, so that the coefficients at frequency $f=f_c$ become those for $f=0$, etc., making use of the periodic property of the coefficients. These relabled coefficients are then complex-

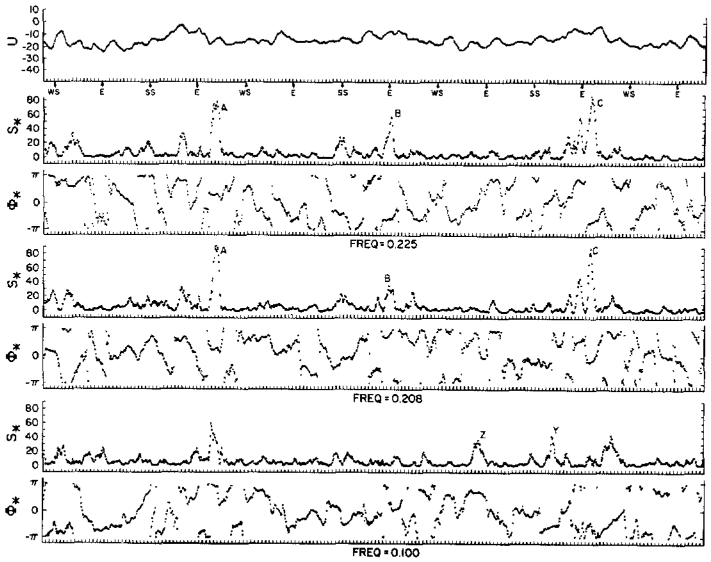

FIGURE 15.—Spectral intensities as a function of time obtained by complex demodulation for the 850-mb v-component at Kwajalein Island. Demodulation was at $f=0.225$, 0.208, and 0.100 day^{-1}. For each, the upper panel, S_*, shows the time variation of the amount of variance in that frequency band (arbitrary units) and the lower panel, Φ_*, the time variation of the phase of the oscillation at that frequency. The topmost panel is a plot of the 850-mb u-component treated with a low-pass filter (in m/s). The time scale (abscissa) is marked in intervals of 1 week and runs from Nov. 30, 1958, through May 12, 1962. A, B, and C designate peaks having intensities exceeding those in the 1/10 per day band; Y and Z, peaks having intensities exceeding those in the 1/4–1/5 per day band.

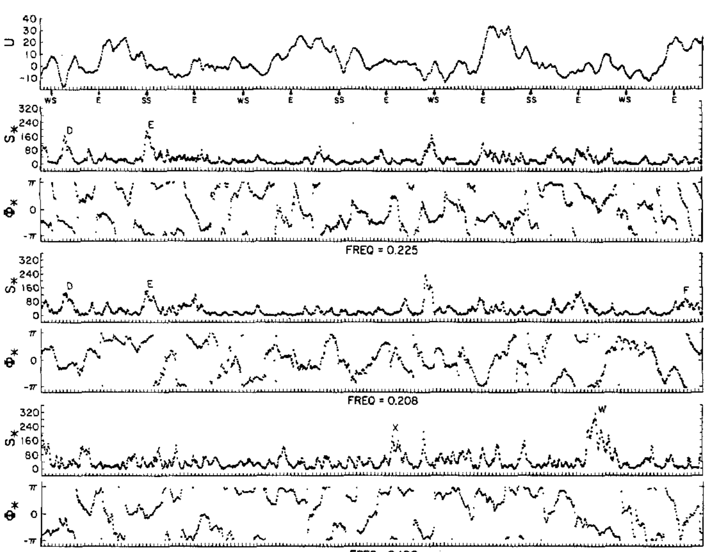

FIGURE 16.—Same as figure 15 except for the Kwajalein Island 200-mb v-component. D, E, X, and W designate peaks having intensities exceeding those in the 1/4–1/5 per day band.

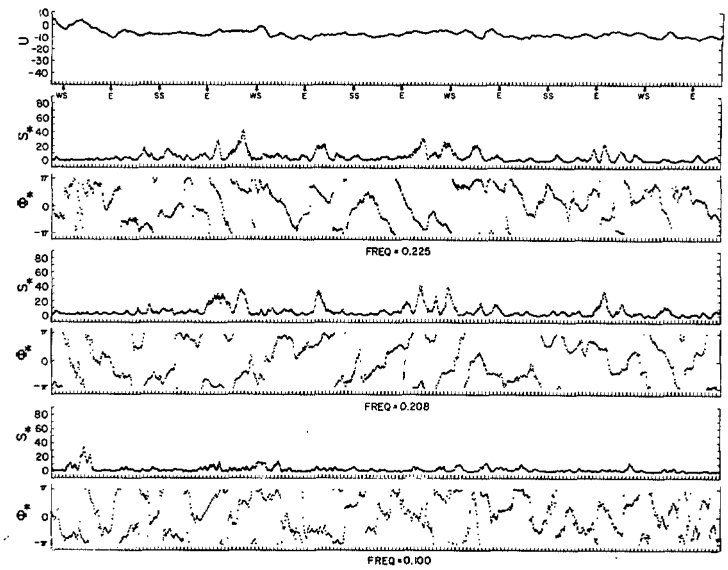

FIGURE 17.—Same as figure 15 except for the Canton Island 850-mb v-component.

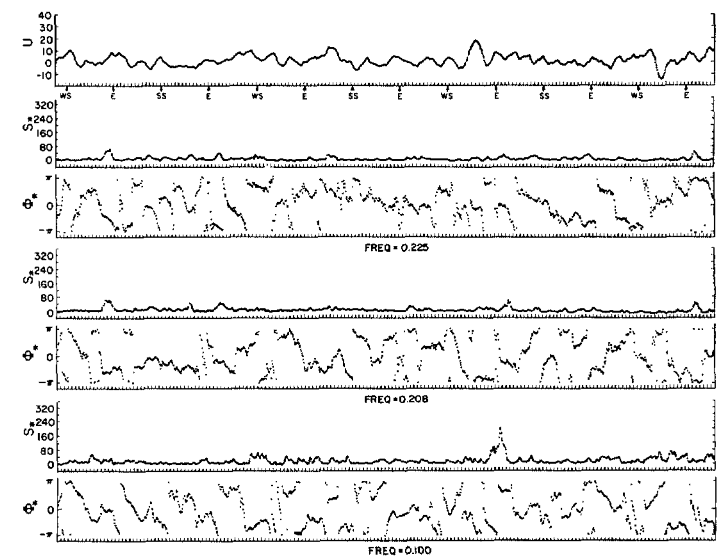

FIGURE 18.—Same as figure 15 except for the Canton Island 200-mb v-component.

multiplied by the appropriate Fourier coefficients of the low-pass filter weights, $w(l)$, and the resulting coefficients used in an inverse fFt. The complex demodulates result.

Because of the possibility that the 1/4–1/5 per day frequency band feature in the spectra of tropical meridional wind components represents a sporadic or nonstationary phenomenon, complex demodulation of the 850- and 200-mb v-series at Canton Island and Kwajalein was performed. Demodulation was performed at three frequencies, $f_c=0.225$, 0.208, and 0.100 per day with a bandwidth identical to that used in the spectral analyses, 0.0164 per day.

Figures 15 and 16 show the 850- and 200-mb v-series for Kwajalein and figures 17 and 18 similarly for Canton Island. The time interval spanned is Nov. 30, 1958, through May 12, 1962. The topmost panel in each figure is a low-pass filter-smoothed series of the zonal wind component at the respective level. The low-pass filter used eliminates fluctuations with frequencies greater than 1/21 per day. The ordinate scales for the spectral and phase demodulates, $S_*(t, f_c)$ and $\Phi_*(t, f_c)$, are the same for the three frequencies at the same pressure level to facilitate comparison of the spectral intensities. The time scale is divided into weeks with the equinoxes and solstices marked.

The following features of these figures are of immediate interest:

1. The $S_*(t,f_c)$ for both 850- and 200-mb v at the frequencies $f_c=0.225$ and 0.208 per day are positively correlated, but the series for $f_c=0.100$ per day is only weakly correlated, if at all, with these two series. That is, maximum values of the spectral intensities at the two higher frequencies tend to occur during the same time intervals with no apparent tendency for these intervals to coincide with the times of maximum values of $S_*(t,f_c=0.100)$. The conclusion to be drawn from this behavior is that the time dependent behavior of the meridional wind spectra is characterized by a relatively broad frequency-band fluctuation in the 1/4–1/5 per day region that does not extend, however, to half that frequency. Moreover, these data do not indicate that the entire spectral curve shifts to larger or smaller ordinate values simultaneously.

2. At 850 mb, there is a pronounced tendency for the 1/4–1/5 per day frequency band to have maximum spectral intensities in the season between the Northern Hemisphere autumnal equinox and winter solstice. Table 2 presents a summary of months in which distinct peaks at both $f_c=0.225$ and 0.208 per day occur at 850 mb. Two 3½-yr intervals for both Canton and Kwajalein are summarized in the table (only one period, that of figs. 15–18, is presented in graphical form, however). The five most intense peaks in the intervals were tabulated. The preferred time of year for the 4–5-day period spectral maxima is quite obviously October–November.

3. Also quite apparent in the figures is the tendency, during limited time intervals, for the spectral intensities in the 1/4–1/5 per day frequency band to exceed those in the 1/10 per day band. As examples, three peaks are marked A, B, and C in figure 15. At 850 mb, the average value of the spectral intensities in the two higher frequency bands must, of course, be greater than that of the 1/10 per day band—this is clearly seen by reference to the spectra themselves (fig. 4). However, there are intervals when the spectral intensities in the lower frequency band exceed those in the 1/4–1/5 per day band. Intervals marked by Y and Z (fig. 15) are examples. Such behavior also holds at 200 mb, for example, in the intervals marked D, E, X, and W (fig. 16), respectively. Thus, a chance or deliberate selection of a few months' data at an appropriate time could result in a sample spectrum which possesses a maximum at a 4–5-day period, a 10-day period, or presumably any other period, in spite of the failure of such a feature to be retained in the spectrum computed over a much longer time interval. The behavior of the spectral demodulates of the meridional wind shown here illustrates the special problem in the application of significance theory to sample meteorological spectra: if the behavior is what can be expected of a stationary time series, then existing sampling theory is correct and adequate. If not, then the problem of statistical significance of sample spectra is not resolvable with current methods. Tests as to whether or not the behavior of the spectral demodulates shown in figures 15–18 can be considered as resulting from a stationary process are beyond the scope of the present paper. Some preliminary comparisons using pseudorandom numbers suggest that the observed behavior cannot be so considered.

4. There is no observable tendency for the maximum spectral intensities at any of the three frequencies to occur simultaneously at 850 and 200 mb. There is, however, a tendency for maximum spectral intensities at 850 mb in the 1/4–1/5 per day band to occur simultaneously at Canton and Kwajalein, or, with a slight lag, at Canton. Although this is a subjective assessment, the basis for it may be seen from comparing figures 15 and 17 or the first column in table 2. In section 3, the uncertain significance of the values of the coherence-square values between the 850- and 200-mb v winds for this frequency interval was noted. If the relative maxima in coherence square in figure 8 are to be regarded as significant, then the relationship in the meridional components in the lower and upper troposphere at these stations must be of a rather peculiar nature because there is little or no tendency for the 4–5-day waves at the two levels to be well developed simultaneously. To investigate thoroughly the time variation of this coherence and relate that behavior to the variation of the variance of the meridional wind

Table 2.—*Summary of months during which the 1/4–1/5 per day frequency band exhibited maximum intensities (850 mb)*

Kwajalein (fig. 15)	
Nov. 1958–May 1962	Mar. 1954–Sept. 1957
1. Oct.–Nov. 1959 (A)*	1. May 1955
2. Sept.–Oct. 1961 (C)*	2. June–July 1955
3. Sept.–Oct. 1960 (B)*	3. Oct. 1956
4. Jan.–Feb. 1959	4. Dec.–Jan. 1955–56
5. Aug. 1959	5. Aug. 1955
Canton (fig. 17)	
Nov. 1958–May 1962	Apr. 1962–Oct. 1965
1. Nov.–Dec. 1959 (A?)*	1. June 1963
2. Oct. 1959	2. Nov.–Dec. 1962
3. Oct.–Dec. 1960 (B?)*	3. Nov. 1964
4. Apr. 1960	4. Oct. 1962
5. Oct. 1961 (C?)*	5. Oct. 1963

*Peaks having intensities exceeding those in the 1/10 per day band.

itself, one should obtain the cospectral demodulates of the 850- and 200-mb v wind. This is beyond the scope of the present paper, but will be accomplished.

5. There does not appear to be any correspondence between the sporadic occurrences of large spectral estimates at any of the three frequencies and either positive or negative simultaneous deviations of the zonal wind.

6. The phase information contained in the plots of Φ_* does not contribute directly to the previous discussion and is included for completeness. We would not expect wave motion in the atmosphere to be phase-coherent or phase-locked in the sense that Φ_* tends to a particular value whenever the waves about frequency f_c are strong, and indeed there is no evidence for this in figures 15–18. The phase demodulates are useful here in one sense and that is they allow a qualitative estimate of how closely tuned the frequency of demodulation f_c is to the center frequency of the local maximum in the spectrum (Granger and Hatanaka 1964). For example, the peak labeled A in figure 15 has associated with it constant Φ_* values for both $f_c=0.225$ and 0.208 per day, indicating that the spectrum has a true broad-band behavior in that frequency range during that time interval. On the other hand, peak W, figure 16, shows a regular trend in the Φ_* values throughout its occurrence. This phase slippage indicates that $f_c=0.100$ per day is not in the center of the actual spectral peak during this interval but is detuned slightly.

6. CONCLUSIONS

I have no intention of inferring that the utility of spectral analysis for the investigation of wave motion in the atmosphere is restricted to wind component data alone. Indeed, cross-spectral analysis of wind components at various stations and levels and between the zonal and meridional component at the same level, as well as multivariate analyses using pressure and temperature data together with wind data, should all be used for a thorough investigation. I have not attempted these analyses here.

However, if the organized motion or wave mode hypothesized is evident in mutual relationships between wind components, pressure, temperature, and mixing ratio, but is not found as some spectral feature on the individual power or variance spectra, then some special consideration must certainly apply. For example, the 850–200-mb meridional component coherence values suggest a coupling in the 1/4–1/5 per day frequency band at Kwajalein and Canton, but no peaks are evident in that band on the individual 200-mb v-spectra. Further statistical investigation as well as theoretical work seems called for in this instance.

A number of considerations must be taken into account when the complete analysis is performed. If some a priori basis exists for specifying the structure of the wave disturbance in the horizontal and vertical and the horizontal momentum and enthalpy transport to be expected, then the use of spectral techniques is straightforward. On the other hand, if spectral analysis is extended to other variables and to include cross-spectral analysis on a heuristic or diagnostic basis that is largely or wholly posteriori, then serious problems will inevitably arise. The problem of the statistical significance becomes serious—the more quasi-independent variables or series there are involved in the analysis, the more likely "significant" results will be obtained that in reality are merely sampling fluctuations. The task of assigning significance levels in such a compounding of analyses quickly becomes an impossible one. Another problem involving a statistical consideration which is almost impossible to treat is that of preselection of the sample to be subjected to analysis. If there is an element of selection involved, the investigator ipso facto had some reason for doing so. In this instance, it is not clear if any choice of significance level, a priori or posteriori, is proper.

I have tried to point out in this contribution that the effective use of spectrum analysis in establishing that wave modes or specific motion systems exist in the atmosphere is not always a simple matter. Dominant spatial scales which clearly exist in the meridional component of transient upper tropospheric winds both in middle latitudes and in the Tropics do not have counterpart temporal scales. At least some features of the spectra of tropical wind components appear to be nonstationary—either the spectrum is time dependent or spectral features are sporadic in nature. Evidence for a tuned wave mode producing a spectral peak seems clear only for the lower troposphere in the Pacific region.

APPENDIX

METHOD OF ESTIMATING THE SPECTRA AND CROSS-SPECTRA USING THE FAST FOURIER TRANSFORM

The method or algorithm used to estimate all of the spectra shown in this paper employs the fast Fourier transform (fFt) and the smoothed periodogram. A sketchy description of this method is included in the discussion of Bingham et al. (1967). The advantages of using the fFt in spectral estimation can be appreciated by noting that many arithmetic operations involving convolution, digital filtering, and autocovariance calculations as examples are conveniently accomplished by using the Fourier coefficients of a sample function. These advantages are, in a sense, a bonus resulting from the practicality of performing the fFt in relatively short intervals of computing time.

The algorithm used consists of the following steps:

1. Removal of the sample mean from each value of the time series,

2. Multiplication of the first and last 10 percent (approximately) of the record by a segment of a cosine curve according to

$$v'(t)=v(t)\,\frac{1}{2}\left\{1-\cos\left[\frac{(t-1)\pi}{0.10N}\right]\right\} \quad 1\leq t\leq 0.10N$$

and (5)

$$v'(t)=v(t)\,\frac{1}{2}\left\{1-\cos\left[\frac{(N-t)\pi}{0.10N}\right]\right\} \quad 0.90N\leq t\leq N.$$

This "tapering" operation is essential in reducing leakage of the periodogram spectral window (Bingham et al. 1967); it obviously insures that the end points of the record are equal and that the resulting property of strict periodicity is present.

3. The tapered series is subjected to the fFt transform, and the Fourier coefficients are used to form

$$\hat{I}(\omega_n)=\frac{N^2}{2\pi}\,|A(n)|^2 \tag{6}$$

where

$$\omega_n=\frac{2\pi n}{N} \quad n=0,1,\ .\ .\ .,\ N/2.$$

In this notation, $A(n)$ are the $N/2$ Fourier amplitudes, at corresponding angular frequencies ω_n of the Fourier transform. The $\hat{I}(\omega_n)$s are termed the modified periodogram.

4. The modified periodogram estimates are smoothed with a digital block averaging scheme of length L

$$\hat{S}(\omega_n)=\sum_{l=-L/2}^{L/2} H(l)\hat{I}(\omega_{n-l}). \tag{7}$$

The weights $H(l)$ are chosen to sum to unity and were initially those of a simple block (equally weighted) averaging scheme of length L_1. To produce a smoother spectrum, this averaging was followed by a similar one of length $L_2(L_2<L_1)$. The ratio L_2/L_1 is determined by the response function of the equal-weighting averaging scheme. For details of this averaging scheme sequence see pp. 143–144 of Granger and Hatanaka (1964).

The following characteristics of this estimation scheme should be noted. The scheme results in a spectral window of rectangular shape with a bandwidth equal to $(2L_1/N)\,f_{Ny}$ where f_{Ny} is the Nyquist frequency. Because the modified periodogram contains $N/2$ estimates over the resolvable frequency range $f=0$ to f_{Ny} and are periodic, the final spectral estimates also number $N/2$. Not all $N/2$ of the $\hat{S}(\omega_n)$ estimates are independent, however. If we consider that each modified periodogram estimate has two degrees of freedom, the rectangular spectral window of length L_1 will result in $2L_1$ degrees of freedom for each $\hat{S}(\omega_n)$ estimate. An adjustment is made in the calculation of the degrees of freedom because the tapering operation, eq (5), results in fewer than N available degrees of freedom initially. Integration of the "cosine bells" of eq (5) indicates that N_{eff}, an effective N, is approximately equal to $0.87N$, Therefore,

$$\nu=\frac{2\cdot L_1\cdot N_{eff}}{N}=1.74L_1.$$

Although the fact that the averaging scheme used [or weights $H(l)$ in eq (7)] does not represent exactly a rectangular window, experimentation with spectra calculated with pseudorandom numbers indicates that the spectral estimates are not perceptably different from what would be expected with $1.74L_1$ degrees of freedom.

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Roy Jenne and Dennis Joseph of the National Center for Atmospheric Research Computing Facility were instrumental in collecting the data used in this work. I thank R. A. Madden for advice, critical discussion, and reading the manuscript.

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